# **Lesson Objectives**

1. Apply the Fundamental Counting Principle (FCP) for independent events.
2. Consider restrictions/conditions when using FCP.
3. Evaluate permutations or combinations using graphing calculator.
4. Key words associated with permutations or combinations.
5. Solve problems involving permutations or combinations.

* **(Definition)** Two events are **independent** if neither event influences the outcome of the other.

# The **Fundamental Counting Principle** (**FCP**)

When there are **m** ways to do one thing and **n** ways to do another,   
there are **m × n** ways of doing **both**.

NOTE: The FCP easily works with more than two events as well.

* **Example:** In how many ways can you answer the questions on an exam that consists of 7 multiple choice questions, each of which has 4 answer choices, followed by 5 true-false questions? [8.3-3]

For the first 7 multiple choice questions, each having 4 answer choices, that’s

4 ∙ 4 ∙ 4 ∙ 4 ∙ 4 ∙ 4 ∙ 4 = 47

and for the last 5 true-false questions (2 choices each), that’s

2 ∙ 2 ∙ 2 ∙ 2 ∙ 2 = 25

So, using FCP, there are 47 ∙ 25 = **524,288** ways you can do that.

* **Example:** How many automobile license plates can be made involving 2 letters followed by either 3 or 4 digits? [8.3-4]

We’re assuming that letters and digits can be used more than once. We need to do 2 separate calculations, using FCP for each one. Then we will total them.

* + Case 1: L L D D D, which is 26 ∙ 26 ∙ 10 ∙ 10 ∙ 10 = 262 ∙ 103
  + Case 2: L L D D D D, which is 26 ∙ 26 ∙ 10 ∙ 10 ∙ 10 ∙ 10 = 262 ∙ 104

The total is 262 ∙ 103 + 262 ∙ 104 = **7,436,000** possible license plates that can be made like this.

## Be careful with **RESTRICTIONS** or **CONDITIONS** imposed when using the FCP.

* **(Definition)** When the outcome of one event affects the outcome of another event, they are called **dependent events**. This sometimes happens with FCP.

A common situation with dependent events is where **repetition** is **not** allowed.

* **Example:** How many automobile license plates can be made involving 3 letters followed by 3 digits, if letters cannot be repeated (used more than once) but digits can be repeated? [8.3-8]

Since letters cannot repeat, the second letter **depends** on what the first is, and the third letter **depends** on what the first and second letters are. We need to reduce the number of letters available each time by one:

* + License plate format is: **L L L D D D**
  + Letters can’t repeat, so L L L means **26 ∙ 25 ∙ 24**
  + Digits can repeat, so DDD means **10 ∙ 10 ∙ 10** = **103**
  + Using FCP, there are 26 ∙ 25 ∙ 24 ∙ 103 = **15,600,000** possible license plates.

# Counting Techniques Involving **Dependent** Events (**no repetition**)

* **(Definition)** The **factorial** of a natural number is the product of that number and all the natural numbers smaller than it. (NOTE: 0! is defined to equal 1.)

Simply put, you multiply down, reducing by 1 each time, until you get to 1.

* **Example:** Simplify. 5! [8.3.27]

5! is read as “**Five factorial**,” and means **5 ∙ 4 ∙ 3 ∙ 2 ∙ 1** = **120**

Context problem: How many ways can you arrange 5 different books on a shelf?

Context problem: How many ways can 5 people stand in line (or seated in a row)?

Context problem: How many ways can 5 people compete and finish in a race?

All of those above are solved using the calculation of “Five factorial,” 5! = 120.

This can be done on the **calculator** by pressing:

**5, then MATH, (go to PRB), (choose 4: !), ENTER**.

screenshot of what keys to press on calculator:
5, MATH, left arrow, 4, ENTER displays screenshot of calculating "five factorial," or 5!, with output of 120.

## **Permutation** – order (arrangement or sequencing) matters

A permutation is like a truncated (cut-off) factorial. More on that later. First, let’s look at its notation.

### **Notation** (format) used for **Permutation**:

* P(*n*,*r*) is used in MyMathLab and other textbooks. Example: P(6,2)
* *n* *P* *r* is also found in in textbooks and TI-84 calculator. Example: 6 P 2
* *n* nPr *r* is how it looks on the TI-83/82/81 calculator. Example: 6 nPr 2

### **Formula for Permutation – but there’s an even easier way. (Stay tuned)**

P(*n*,*r*) = You may see this formula introduced in videos or in the Question Help in MyMathLab, but you can IGNORE this formula – there is an easier, faster way on the calculator.

### **What does Permutation mean?**

P(6,2) 6 P 2 6 nPr 2

These all mean “permutation of six things taken 2 at a time.”

* *n* – is the total number available
* *r* – is the size of the grouping
* P(6,2) literally means start with 6 and multiply down like a factorial, reducing by one, but stop after 2 positions.
* **Example:** Evaluate the expression. P(6,2) [8.3.31]

Calculator:press **6, then MATH, (go to PRB), (choose 2: nPr), press 2, ENTER**

 

* **Example:** Context problem for P(6,2)

How many different two-letter codes are there if only the letters A, B, C, D, E, and F can be used and no letter can be used more than once? [8.3.41]

* + Is repetition allowed? NO – the problem states this restriction
  + Does order matter? YES – code AB is different from code BA.
    - Use **permutation** P(*n*,*r*).
  + Total available? ***n* = 6**
  + Size of grouping? ***r* = 2**

P(6,2) = 6 ∙ 5 = 30 (calculator 6 nPr 2) There are **30** different letter codes.

### **Key Words or Situations** for **Permutations** – **order matters**

* **Arrangement** (Arrange)
* **Codes or Passwords** (note the previous example)
* **Officers** of a club – President, Vice-President, Secretary, Treasurer
* **Race/Competition** (order or ranking) – First, Second, Third, etc.
* **Example:** How many ways can a president, vice-president, secretary, and treasurer be chosen from a club with 9 members? [8.3-11]
  + Is repetition allowed? NO – Assume one person cannot hold 2 different offices
  + Does order matter? YES – Amy (Pres), Bill (VP) is different from Bill (Pres), Amy (VP)
    - Use **Permutation** P(*n*,*r*)
  + Total available? ***n* = 9** (there are 9 club members)
  + Size of group? ***r* = 4** (there are 4 officers: Pres, VP, Sec, Treas)

P(9,4) = 9 ∙ 8 ∙ 7 ∙ 6 = 3024 (calculator 9 nPr 4) There are **3024** ways for 4 officers.

## **Combination** – order does **NOT** matter

In a **combination**, all the duplicates are removed. More on that later.

### **Notation (format) used for Combination:**

* C(*n*,*r*) is used in MyMathLab and other textbooks. Example: C(8,3)
* *n* *C* *r* is also found in in textbooks and TI-84 calculator. Example: 8 C 3
* *n* nCr *r* is how it looks on the TI-83/82/81 calculator. Example: 8 nCr 3

### **Formula for Combination – but there’s an even easier way. (Stay tuned)**

C(*n*,*r*) = You may see this formula introduced in videos or in the Question Help in MyMathLab, but you can IGNORE this formula – there is an easier, faster way on the calculator.

Compare the formula C(*n*,*r*) = with the formula P(*n*,*r*) = .

How are they different? The denominator has an extra *r*! multiplied to the (*n* – *r* )!.

This extra denominator factor divides out all the duplicates, indicating that order doesn’t matter.

### **What does Combination mean?**

C(8,3) or 8 C 3 or 8 nCr 3

These all mean “combination of eight things taken 3 at a time.”

* *n* – is the total number available
* *r* – is the size of the grouping
* this is not easily done by hand – please use calculator!
* **Example:** Evaluate the expression. C(8,3) [8.3.59]

Calculator: press **8, then MATH, (go to PRB), (choose 3: nCr), press 3, ENTER**

screenshot of what keys to press on calculator:
8, MATH, left arrow, 3, 3, ENTER screenshot of calculator for combination of eight things taken three at a time, or 8 nCr 3, with output of 56.


### What is the **difference between Combination and Permutation**? Why are duplicates removed?

Let’s consider an example where both the total available (*n*) and the size of the grouping (*r*) are each 3. Suppose we have three fellas: Al, Bill, and Chuck.

* How can these 3 fellas (Al, Bill, and Chuck) be seated in a row of 3 chairs?

A B C A C B B A C B C A C A B C B A

6 total ways – Where they are specifically seated in the row is significant. This is a permutation because the order matters. (calculator 3 nPr 3 or 3P3 , which equals 6)

Now take these same 3 fellas: Al, Bill, and Chuck and change the problem/situation.

* How many ways can these 3 fellas (Al, Bill, and Chuck) stand together in an elevator?

Order doesn’t matter! ABC, ACB, BAC, BCA, CAB, CBA all represent the same 3 fellas in the elevator. So, instead of counting it as 6 separate ways, the five duplicates are discarded. Instead of 6 ways as a permutation, it’s only **one** way as a combination. (calculator 3 nCr 3 or 3C3 , which equals 1)

There are always **far fewer combinations** than permutations, assuming you’re using the same values for *n* and r.

* **Example:** Context problem for C(8,3)

In how many ways can a committee of 3 students be formed from a pool of 8 students? [8.3.68]

* + Is repetition allowed? NO – the same person cannot be duplicated in a group!
  + Does order matter? NO – a committee has no order or special arrangement to it.
    - Use **Combination** C(*n*,*r*)
  + Total available? n = 8 (there is a pool of 8 students)
  + Size of group? r = 3 (the size of the committee is 3)

C(8,3) = use calculator (see previous example) = 8 nCr 3 = 56. There are **56** ways.

### **Keywords or Situations** for **Combinations** – order does **NOT** matter

* (look for anything generic, vague, nondescript – such that no particular order, arrangement, sequence is indicated)
* **Collection or Group/grouping** (note the previous example)
* **Committee** or **team** of people, including a **jury** (note example earlier)
* Chance – **Card Hands** or **Lotteries**
* **Example:** How many 3 card hands are possible with a 26-card deck? [8.3.72]
  + Is repetition allowed? NO – No duplicates of the same card in a hand
  + Does order matter? NO – how you arrange the cards in your hand doesn’t matter; you still have the same three cards.
    - Use **Combination** C(*n*,*r*)
  + Total available? ***n* = 26** (it’s a 26-card deck)
  + Size of group? ***r* = 3** (you have a 3-card hand)

C(26,3) = (use calculator) = 26 nCr3 = 2600. There are **2600** possible 3-card hands.

Sources used:

1. Math is Fun website, with content about the Basic Counting Principle, located at <https://www.mathsisfun.com/data/basic-counting-principle.html>
2. Pearson MyMathLab *College Algebra with Modeling and Visualization, 6th Edition*, Rockswold
3. Wabbitemu calculator emulator version 1.9.5.21 by Revolution Software, BootFree ©2006-2014 Ben Moody, Rom8x ©2005-2014 Andree Chea. Website <https://archive.codeplex.com/?p=wabbit>